Chapter 2. Motion in a Straight Line

1. Preeti reached the metro station and found that the escalator was not working. She walked up the stationary escalator in time t_1. On other days, if she remains stationary on the moving escalator, then the escalator takes her up in time t_2. The time taken by her to walk up on the moving escalator will be

(a) $\frac{t_1t_2}{t_2-t_1}$ (b) $\frac{t_1t_2}{t_2+t_1}$

(c) t_1-t_2 (d) $\frac{t_1+t_2}{2}$

(NEET 2017)

2. Two cars P and Q start from a point at the same time in a straight line and their positions are represented by $x_P(t) = (at + bt^2)$ and $x_Q(t) = (ft - t^2)$. At what time do the cars have the same velocity?

(a) $\frac{a-f}{1+b}$ (b) $\frac{a+f}{2(b-1)}$

(c) $\frac{a+f}{2(1+b)}$ (d) $\frac{f-a}{2(1+b)}$

(NEET-II 2016)

3. If the velocity of a particle is $v = At + Bt^2$, where A and B are constants, then the distance travelled by it between 1 s and 2 s is

(a) $\frac{3}{2}A+\frac{7}{3}B$ (b) $\frac{A}{2}+\frac{B}{3}$

(c) $\frac{3}{2}A+4B$ (d) $3A+7B$

(NEET-I 2016)

4. A particle of unit mass undergoes one-dimensional motion such that its velocity varies according to $v(x) = \beta x^{-2n}$, where β and n are constants and x is the position of the particle. The acceleration of the particle as a function of x, is given by

(a) $-2\beta^2 x^{-2n+1}$ (b) $-2n\beta^2 e^{-4n+1}$

(c) $-2n\beta^2 x^{-2n-1}$ (d) $-2n\beta^2 x^{-4n-1}$

(2015 Cancelled)

5. A stone falls freely under gravity. It covers distances h_1, h_2 and h_3 in the first 5 seconds, the next 5 seconds and the next 5 seconds respectively. The relation between h_1, h_2 and h_3 is

(a) $h_2 = 3h_1$ and $h_3 = 3h_2$

(b) $h_1 = h_2 = h_3$

(c) $h_1 = 2h_2 = 3h_3$

(d) $h_1 = \frac{h_2}{3} = \frac{h_3}{5}$ *(NEET 2013)*

6. The displacement 'x' (in meter) of a particle of mass 'm' (in kg) moving in one dimension under the action of a force, is related to time 't' (in sec) by $t = \sqrt{x}+3$. The displacement of the particle when its velocity is zero, will be

(a) 4 m (b) 0 m (zero)

(c) 6 m (d) 2 m

(Karnataka NEET 2013)

7. The motion of a particle along a straight line is described by equation $x = 8 + 12t - t^3$ where x is in metre and t in second. The retardation of the particle when its velocity becomes zero is

(a) 24 m s^{-2} (b) zero

(c) 6 m s^{-2} (d) 12 m s^{-2} (2012)

8. A boy standing at the top of a tower of 20 m height drops a stone. Assuming g = 10 m s^{-2}, the velocity with which it hits the ground is

(a) 10.0 m/s (b) 20.0 m/s

(c) 40.0 m/s (d) 5.0 m/s *(2011)*

9. A particle covers half of its total distance with speed v_1 and the rest half distance with speed v_2. Its average speed during the complete journey is

(a) $\frac{v_1+v_2}{2}$ (b) $\frac{v_1v_2}{v_1+v_2}$

(c) $\frac{2v_1v_2}{v_1+v_2}$ (d) $\frac{v_1^2v_2^2}{v_1^2+v_2^2}$

(Mains 2011)

10. A particle moves a distance x in time t according to equation $x = (t + 5)^{-1}$. The acceleration of particle is proportional to
(a) $(\text{velocity})^{3/2}$ (b) $(\text{distance})^2$
(c) $(\text{distance})^{-2}$ (d) $(\text{velocity})^{2/3}$ *(2010)*

11. A ball is dropped from a high rise platform at $t = 0$ starting from rest. After 6 seconds another ball is thrown downwards from the same platform with a speed v. The two balls meet at $t = 18$ s. What is the value of v?
(Take $g = 10$ m/s^2)
(a) 75 m/s (b) 55 m/s
(c) 40 m/s (d) 60 m/s *(2010)*

12. A particle starts its motion from rest under the action of a constant force. If the distance covered in first 10 seconds is S_1 and that covered in the first 20 seconds is S_2, then
(a) $S_2 = 3S_1$ (b) $S_2 = 4S_1$
(c) $S_2 = S_1$ (d) $S_2 = 2S_1$ *(2009)*

13. A bus is moving with a speed of 10 ms^{-1} on a straight road. A scooterist wishes to overtake the bus in 100 s. If the bus is at a distance of 1 km from the scooterist, with what speed should the scooterist chase the bus?
(a) 40 m s^{-1} (b) 25 m s^{-1}
(c) 10 m s^{-1} (d) 20 m s^{-1} *(2009)*

14. A particle moves in a straight line with a constant acceleration. It changes its velocity from 10 ms^{-1} to 20 ms^{-1} while passing through a distance 135 m in t second. The value of t is
(a) 12 (b) 9
(c) 10 (d) 1.8 *(2008)*

15. The distance travelled by a particle starting from rest and moving with an acceleration $\frac{4}{3}$ m s^{-2}, in the third second is
(a) $\frac{10}{3}$ m (b) $\frac{19}{3}$ m
(c) 6 m (d) 4 m *(2008)*

16. A particle moving along x-axis has acceleration f, at time t, given by $f = f_0\left(1 - \frac{t}{T}\right)$, where f_0 and T are constants. The particle at $t = 0$ has zero velocity. In the time interval between $t = 0$ and the instant when $f = 0$, the particle's velocity (v_x) is
(a) $\frac{1}{2}f_0T^2$ (b) f_0T^2
(c) $\frac{1}{2}f_0T$ (d) f_0T. *(2007)*

17. A car moves from X to Y with a uniform speed v_u and returns to Y with a uniform speed v_d. The average speed for this round trip is
(a) $\sqrt{v_u v_d}$ (b) $\frac{v_d v_u}{v_d + v_u}$
(c) $\frac{v_u + v_d}{2}$ (d) $\frac{2v_d v_u}{v_d + v_u}$. *(2007)*

18. The position x of a particle with respect to time t along x-axis is given by $x = 9t^2 - t^3$ where x is in metres and t in seconds. What will be the position of this particle when it achieves maximum speed along the $+x$ direction?
(a) 54 m (b) 81 m
(c) 24 m (d) 32 m. *(2007)*

19. Two bodies A (of mass 1 kg) and B (of mass 3 kg) are dropped from heights of 16 m and 25 m, respectively. The ratio of the time taken by them to reach the ground is
(a) 4/5 (b) 5/4
(c) 12/5 (d) 5/12. *(2006)*

20. A car runs at a constant speed on a circular track of radius 100 m, taking 62.8 seconds for every circular lap. The average velocity and average speed for each circular lap respectively is
(a) 10 m/s, 0 (b) 0, 0
(c) 0, 10 m/s (d) 10 m/s, 10 m/s. *(2006)*

21. A particle moves along a straight line OX. At a time t (in seconds) the distance x (in metres) of the particle from O is given by $x = 40 + 12t - t^3$. How long would the particle travel before coming to rest?
(a) 16 m (b) 24 m
(c) 40 m (d) 56 m. *(2006)*

22. A ball is thrown vertically upward. It has a speed of 10 m/sec when it has reached one half of its maximum height. How high does the ball rise?
(Take $g = 10$ m/s^2.)
(a) 10 m (b) 5 m
(c) 15 m (d) 20 m. *(2005)*

23. The displacement x of a particle varies with time t as $x = ae^{-\alpha t} + be^{\beta t}$, where a, b, α and β are positive constants. The velocity of the particle will
(a) be independent of β
(b) drop to zero when $\alpha = \beta$
(c) go on decreasing with time
(d) go on increasing with time. *(2005)*

24. A man throws balls with the same speed vertically upwards one after the other at an interval of 2 seconds. What should be the speed of the throw so that more than two balls are in the sky at any time ? (Given $g = 9.8$ m/s^2)
(a) more than 19.6 m/s (b) at least 9.8 m/s
(c) any speed less than 19.6 m/s
(d) only with speed 19.6 m/s. *(2003)*

25. If a ball is thrown vertically upwards with speed u, the distance covered during the last t seconds of its ascent is
(a) ut (b) $\frac{1}{2}gt^2$
(c) $ut - \frac{1}{2}gt^2$ (d) $(u + gt)\,t$. *(2003)*

26. A particle is thrown vertically upward. Its velocity at half of the height is 10 m/s, then the maximum height attained by it ($g = 10$ m/s^2)
(a) 8 m (b) 20 m
(c) 10 m (d) 16 m. *(2001)*

27. Motion of a particle is given by equation $s = (3t^3 + 7t^2 + 14t + 8)$ m. The value of acceleration of the particle at $t = 1$ sec is
(a) 10 m/s^2 (b) 32 m/s^2
(c) 23 m/s^2 (d) 16 m/s^2. *(2000)*

28. A car moving with a speed of 40 km/h can be stopped by applying brakes after at least 2 m. If the same car is moving with a speed of 80 km/h, what is the minimum stopping distance?
(a) 4 m (b) 6 m
(c) 8 m (d) 2 m. *(1998)*

29. A rubber ball is dropped from a height of 5 m on a plane. On bouncing it rises to 1.8 m. The ball loses its velocity on bouncing by a factor of
(a) $\frac{3}{5}$ (b) $\frac{2}{5}$ (c) $\frac{16}{25}$ (d) $\frac{9}{25}$. *(1998)*

30. The position x of a particle varies with time, (t) as $x = at^2 - bt^3$. The acceleration will be zero at time t is equal to
(a) $\frac{a}{3b}$ (b) zero (c) $\frac{2a}{3b}$ (d) $\frac{a}{b}$. *(1997)*

31. If a car at rest accelerates uniformly to a speed of 144 km/h in 20 sec, it covers a distance of
(a) 1440 cm (b) 2980 cm
(c) 20 m (d) 400 m. *(1997)*

32. A body dropped from a height h with initial velocity zero, strikes the ground with a velocity 3 m/s. Another body of same mass dropped from the same height h with an initial velocity of 4 m/s. The final velocity of second mass, with which it strikes the ground is
(a) 5 m/s (b) 12 m/s
(c) 3 m/s (d) 4 m/s. *(1996)*

33. The acceleration of a particle is increasing linearly with time t as bt. The particle starts from origin with an initial velocity v_0. The distance travelled by the particle in time t will be
(a) $v_0 t + \frac{1}{3}bt^2$ (b) $v_0 t + \frac{1}{2}bt^2$
(c) $v_0 t + \frac{1}{6}bt^3$ (d) $v_0 t + \frac{1}{3}bt^3$. *(1995)*

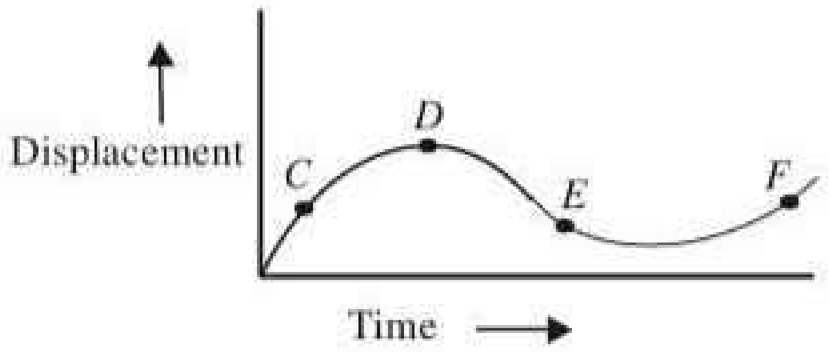
34. The water drop falls at regular intervals from a tap 5 m above the ground. The third drop is leaving the tap at instant the first drop touches the ground. How far above the ground is the second drop at that instant?
(a) 3.75 m (b) 4.00 m
(c) 1.25 m (d) 2.50 m. *(1995)*

35. A car accelerates from rest at a constant rate α for some time after which it decelerates at a constant rate β and comes to rest. If total time elapsed is t, then maximum velocity acquired by car will be
(a) $\frac{(\alpha^2 - \beta^2)t}{\alpha\beta}$ (b) $\frac{(\alpha^2 + \beta^2)t}{\alpha\beta}$
(c) $\frac{(\alpha + \beta)t}{\alpha\beta}$ (d) $\frac{\alpha\beta t}{\alpha + \beta}$. *(1994)*

36. A particle moves along a straight line such that its displacement at any time t is given by $s = (t^3 - 6t^2 + 3t + 4)$ metres. The velocity when the acceleration is zero is
(a) 3 m/s (b) 42 m/s
(c) − 9 m/s (d) −15 m/s. *(1994)*

37. The velocity of train increases uniformly from 20 km/h to 60 km/h in 4 hours. The distance travelled by the train during this period is
(a) 160 km (b) 180 km
(c) 100 km (d) 120 km. *(1994)*

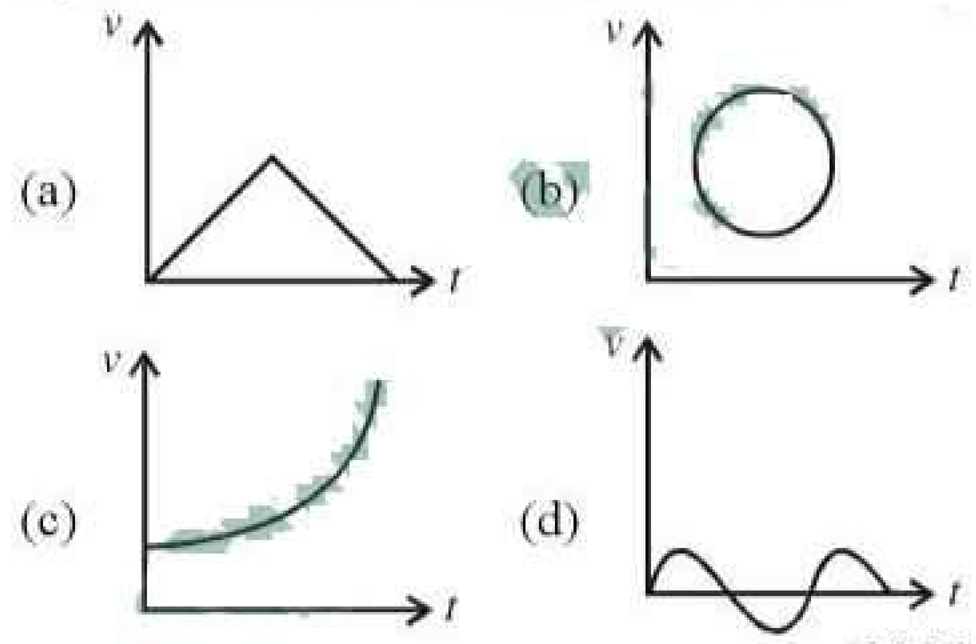
38. The displacement-time graph of a moving particle is shown below. The instantaneous velocity of the particle is negative at the point



(a) E (b) F (c) C (d) D
(1994)

39. A body starts from rest, what is the ratio of the distance travelled by the during the 4th and 3rd second ?
(a) $\frac{7}{5}$ (b) $\frac{5}{7}$
(c) $\frac{7}{3}$ (d) $\frac{3}{7}$. *(1993)*

40. Which of the following curve does not represent motion in one dimension ?



(1992)

41. A body dropped from top of a tower fall through 40 m during the last two seconds of its fall. The height of tower is (g = 10 m/s^2)
(a) 60 m (b) 45 m
(c) 80 m (d) 50 m.
(1992)

42. A car moves a distance of 200 m. It covers the first half of the distance at speed 40 km/h and the second half of distance at speed v. The average speed is 48 km/h. The value of v is
(a) 56 km/h (b) 60 km/h
(c) 50 km/h (d) 48 km/h.
(1991)

43. A bus travelling the first one-third distance at a speed of 10 km/h, the next one-third at 20 km/h and at last one-third at 60 km/h. The average speed of the bus is
(a) 9 km/h (b) 16 km/h
(c) 18 km/h (d) 48 km/h.
(1991)

44. A car covers the first half of the distance between two places at 40 km/h and another half at 60 km/h. The average speed of the car is
(a) 40 km/h (b) 48 km/h
(c) 50 km/h (d) 60 km/h.
(1990)

45. What will be the ratio of the distance moved by a freely falling body from rest in 4th and 5th seconds of journey ?
(a) 4 : 5 (b) 7 : 9
(c) 16 : 25 (d) 1 : 1.
(1989)

46. A car is moving along a straight road with a uniform acceleration. It passes through two points P and Q separated by a distance with velocity 30 km/h and 40 km/h respectively. The velocity of the car midway between P and Q is
(a) 33.3 km/h (b) $20\sqrt{2}$ km/h
(c) $25\sqrt{2}$ km/h (d) 35 km/h.
(1988)

Answer Key

1.	(b)	**2.**	(d)	**3.**	(a)	**4.**	(d)	**5.**	(d)	**6.**	(b)	**7.**	(d)	**8.**	(b)	**9.**	(c)	**10.**	(a)
11.	(a)	**12.**	(b)	**13.**	(d)	**14.**	(b)	**15.**	(a)	**16.**	(c)	**17.**	(d)	**18.**	(a)	**19.**	(a)	**20.**	(c)
21.	(a)	**22.**	(a)	**23.**	(d)	**24.**	(a)	**25.**	(b)	**26.**	(c)	**27.**	(b)	**28.**	(c)	**29.**	(a)	**30.**	(a)
31.	(d)	**32.**	(a)	**33.**	(c)	**34.**	(a)	**35.**	(d)	**36.**	(c)	**37.**	(a)	**38.**	(a)	**39.**	(a)	**40.**	(b)
41.	(b)	**42.**	(b)	**43.**	(c)	**44.**	(b)	**45.**	(b)	**46.**	(c)								

EXPLANATIONS

1. **(b) :** Let v_1 is the velocity of Preeti on stationary escalator and d is the distance travelled by her

$\therefore \quad v_1 = \frac{d}{t_1}$

Again, let v_2 is the velocity of escalator

$\therefore \quad v_2 = \frac{d}{t_2}$

$\therefore$ Net velocity of Preeti on moving escalator with respect to the ground

$$v = v_1 + v_2 = \frac{d}{t_1} + \frac{d}{t_2} = d\left(\frac{t_1 + t_2}{t_1 t_2}\right)$$

The time taken by her to walk up on the moving escalator will be

$$t = \frac{d}{v} = \frac{d}{d\left(\frac{t_1 + t_2}{t_1 t_2}\right)} = \frac{t_1 t_2}{t_1 + t_2}$$

2. **(d) :** Position of the car P at any time t, is

$x_P(t) = at + bt^2$

$$v_P(t) = \frac{dx_P(t)}{dt} = a + 2bt \qquad \text{...(i)}$$

Similarly, for car Q,

$x_Q(t) = ft - t^2$

$$v_Q(t) = \frac{dx_Q(t)}{dt} = f - 2t \qquad \text{...(ii)}$$

$\because \quad v_P(t) = v_Q(t)$ (Given)

$\therefore \quad a + 2bt = f - 2t$ or, $2t(b+1) = f - a$

$\therefore \quad t = \frac{f-a}{2(1+b)}$

3. **(a) :** Velocity of the particle is $v = At + Bt^2$

$\frac{ds}{dt} = At + Bt^2$, $\int ds = \int (At + Bt^2)dt$

$\therefore \quad s = \frac{At^2}{2} + B\frac{t^3}{3} + C$

$s(t = 1\text{s}) = \frac{A}{2} + \frac{B}{3} + C$, $s(t = 2\text{s}) = 2A + \frac{8}{3}B + C$

Required distance $= s(t = 2\text{ s}) - s(t = 1\text{ s})$

$$= \left(2A + \frac{8}{3}B + C\right) - \left(\frac{A}{2} + \frac{B}{3} + C\right) = \frac{3}{2}A + \frac{7}{3}B$$

4. **(d) :** According to question, velocity of unit mass varies as

$$v(x) = \beta x^{-2n} \qquad \text{...(i)}$$

$$\frac{dv}{dx} = -2n\beta x^{-2n-1} \qquad \text{...(ii)}$$

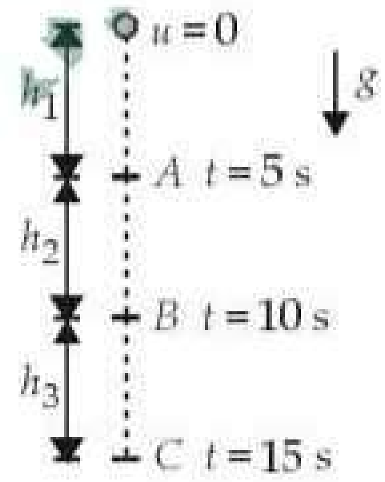
Acceleration of the particle is given by

$$a = \frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt} = \frac{dv}{dx} \times v$$

Using equation (i) and (ii), we get

$$a = (-2n\beta x^{-2n-1}) \times (\beta x^{-2n}) = -2n\beta^2 x^{-4n-1}$$

5. **(d) :** Distance covered by the stone in first 5 seconds



(*i.e.* $t = 5$ s) is

$$h_1 = \frac{1}{2}g(5)^2 = \frac{25}{2}g \qquad \text{...(i)}$$

Distance travelled by the stone in next 5 seconds (*i.e.* $t = 10$ s) is

$$h_1 + h_2 = \frac{1}{2}g(10)^2 = \frac{100}{2}g \qquad \text{...(ii)}$$

Distance travelled by the stone in next 5 seconds (*i.e.* $t = 15$ s) is

$$h_1 + h_2 + h_3 = \frac{1}{2}g(15)^2 = \frac{225}{2}g \qquad \text{...(iii)}$$

Subtract (i) from (ii), we get

$$(h_1 + h_2) - h_1 = \frac{100}{2}g - \frac{25}{2}g = \frac{75}{2}g$$

$$h_2 = \frac{75}{2}g = 3h_1 \qquad \text{...(iv)}$$

Subtract (ii) from (iii), we get

$$(h_1 + h_2 + h_3) - (h_2 + h_1) = \frac{225}{2}g - \frac{100}{2}g$$

$$h_3 = \frac{125}{2}g = 5h_1 \qquad \text{...(v)}$$

From (i), (iv) and (v), we get

$$h_1 = \frac{h_2}{3} = \frac{h_3}{5}$$

6. **(b) :** Given : $t = \sqrt{x} + 3$ or $\sqrt{x} = t - 3$

Squaring both sides, we get

$x = (t-3)^2$

Velocity, $v = \frac{dx}{dt} = \frac{d}{dt}(t-3)^2 = 2(t-3)$

Velocity of the particle becomes zero, when

$2(t-3) = 0$ or $t = 3$ s

At $t = 3$ s,

$x = (3-3)^2 = 0$ m

7. **(d) :** Given : $x = 8 + 12t - t^3$

Velocity, $v = \frac{dx}{dt} = 12 - 3t^2$

When $v = 0$, $12 - 3t^2 = 0$ or $t = 2$ s

$$a = \frac{dv}{dt} = -6t$$

$$a|_{t=2\,s} = -12 \text{ m s}^{-2}$$

Retardation = 12 m s^{-2}

8. **(b) :** Here, $u = 0$, $g = 10$ m s^{-2}, $h = 20$ m

Let v be the velocity with which the stone hits the ground.

$\therefore\ v^2 = u^2 + 2gh$

or $v = \sqrt{2gh} = \sqrt{2 \times 10 \times 20} = 20$ m/s $(\because\ u = 0)$

9. **(c) :** Let S be the total distance travelled by the particle.

Let t_1 be the time taken by the particle to cover first half of the distance. Then

$$t_1 = \frac{S/2}{v_1} = \frac{S}{2v_1}$$

Let t_2 be the time taken by the particle to cover remaining half of the distance. Then

$$t_2 = \frac{S/2}{v_2} = \frac{S}{2v_2}$$

Average speed, $v_{av} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$

$$= \frac{S}{t_1 + t_2} = \frac{S}{\frac{S}{2v_1} + \frac{S}{2v_2}} = \frac{2v_1 v_2}{v_1 + v_2}$$

10. **(a) :** Distance, $x = (t + 5)^{-1}$ ...(i)

Velocity, $v = \frac{dx}{dt} = \frac{d}{dt}(t+5)^{-1} = -(t+5)^{-2}$ ...(ii)

Acceleration

$$a = \frac{dv}{dt} = \frac{d}{dt}[-(t+5)^{-2}] = 2(t+5)^{-3} \quad \text{...(iii)}$$

From equation (ii), we get

$$v^{3/2} = -(t+5)^{-3} \quad \text{...(iv)}$$

Substituting this in equation (iii) we get

Acceleration, $a = -2v^{3/2}$ or $a \propto (\text{velocity})^{3/2}$

From equation (i), we get

$$x^3 = (t+5)^{-3}$$

Substituting this in equation (iii), we get

Acceleration, $a = 2x^3$ or $a \propto (\text{distance})^3$

Hence option (a) is correct.

11. **(a) :** Let the two balls meet after t s at distance x from the platform.

For the first ball

$$u = 0,\ t = 18 \text{ s},\ g = 10 \text{ m/s}^2$$

Using $h = ut + \frac{1}{2}gt^2$

$$\therefore\ x = \frac{1}{2} \times 10 \times 18^2 \quad \text{...(i)}$$

For the second ball

$$u = v,\ t = 12 \text{ s},\ g = 10 \text{ m/s}^2$$

Using $h = ut + \frac{1}{2}gt^2$

$$\therefore\ x = v \times 12 + \frac{1}{2} \times 10 \times 12^2 \quad \text{...(ii)}$$

From equations (i) and (ii), we get

$$\frac{1}{2} \times 10 \times 18^2 = 12v + \frac{1}{2} \times 10 \times (12)^2$$

or $12v = \frac{1}{2} \times 10 \times [(18)^2 - (12)^2]$

$$= \frac{1}{2} \times 10 \times [(18+12)(18-12)]$$

$$12v = \frac{1}{2} \times 10 \times 30 \times 6$$

or $v = \frac{1 \times 10 \times 30 \times 6}{2 \times 12} = 75$ m/s

12. **(b) :** Given $u = 0$.

Distance travelled in 10 s, $S_1 = \frac{1}{2}a \cdot 10^2 = 50a$

Distance travelled in 20 s, $S_2 = \frac{1}{2}a \cdot 20^2 = 200a$

$\therefore\ S_2 = 4S_1$.

13. **(d) :** Let v_s be the velocity of the scooter, the distance between the scooter and the bus = 1000 m,

The velocity of the bus = 10 ms^{-1}

Time taken to overtake = 100 s

Relative velocity of the scooter with respect to the bus = $(v_s - 10)$

$$\therefore\ \frac{1000}{(v_s - 10)} = 100 \text{ s} \Rightarrow v_s = 20 \text{ ms}^{-1}.$$

14. **(b) :** $v^2 - u^2 = 2as$

Given $v = 20$ ms^{-1}, $u = 10$ ms^{-1}, $s = 135$ m

$$\therefore\ a = \frac{400 - 100}{2 \times 135} = \frac{300}{270} = \frac{10}{9} \text{ m/s}^2$$

$$v = u + at \Rightarrow t = \frac{v - u}{a} = \frac{10 \text{ m/s}}{\frac{10}{9} \text{ m/s}^2} = 9 \text{ s}$$

15. **(a) :** Distance travelled in the 3[rd] second

= Distance travelled in 3 s

− distance travelled in 2 s.

As, $u = 0$,

$$S_{(3^{rd}\ s)} = \frac{1}{2}a\cdot 3^2 - \frac{1}{2}a\cdot 2^2 = \frac{1}{2}\cdot a\cdot 5$$

Given $a = \frac{4}{3}\text{ ms}^{-2}$; $\therefore\ S_{(3^{rd}\ s)} = \frac{1}{2}\times\frac{4}{3}\times 5 = \frac{10}{3}\text{ m}$

16. (c) : Given : At time $t = 0$, velocity, $v = 0$.

Acceleration $f = f_0\left(1 - \frac{t}{T}\right)$

At $f = 0$, $0 = f_0\left(1 - \frac{t}{T}\right)$

Since f_0 is a constant,

$\therefore\ 1 - \frac{t}{T} = 0$ or $t = T$.

Also, acceleration $f = \frac{dv}{dt}$

$$\therefore \int_0^{v_x} dv = \int_{t=0}^{t=T} f dt = \int_0^T f_0\left(1 - \frac{t}{T}\right)dt$$

$$\therefore\ v_x = \left[f_0 t - \frac{f_0 t^2}{2T}\right]_0^T = f_0 T - \frac{f_0 T^2}{2T} = \frac{1}{2}f_0 T.$$

17. (d) : Average speed $= \frac{\text{total distance travelled}}{\text{total time taken}}$

$$= \frac{s+s}{t_1+t_2} = \frac{2s}{\frac{s}{v_u}+\frac{s}{v_d}} = \frac{2v_u v_d}{v_d+v_u}$$

18. (a) : Given : $x = 9t^2 - t^3$ ...(i)

Speed $v = \frac{dx}{dt} = \frac{d}{dt}(9t^2 - t^3) = 18t - 3t^2$.

For maximum speed, $\frac{dv}{dt} = 0 \Rightarrow 18 - 6t = 0$

$\therefore\ t = 3$ s.

$\therefore\ x_{max} = 81\text{ m} - 27\text{ m} = 54\text{ m}$. (From $x = 9t^2 - t^3$)

19. (a) : Time taken by a body fall from a height h to reach the ground is $t = \sqrt{\frac{2h}{g}}$.

$$\therefore\ \frac{t_A}{t_B} = \frac{\sqrt{\frac{2h_A}{g}}}{\sqrt{\frac{2h_B}{g}}} = \sqrt{\frac{h_A}{h_B}} = \sqrt{\frac{16}{25}} = \frac{4}{5}.$$

20. (c) : Distance travelled in one rotation (lap) $= 2\pi r$

$\therefore$ Average speed $= \frac{\text{distance}}{\text{time}} = \frac{2\pi r}{t}$

$$= \frac{2\times 3.14\times 100}{62.8} = 10\text{ m s}^{-1}$$

Net displacement in one lap = 0

Average velocity $= \frac{\text{net displacement}}{\text{time}} = \frac{0}{t} = 0.$

21. (a) : $x = 40 + 12t - t^3$

$\therefore$ Velocity $v = \frac{dx}{dt} = 12 - 3t^2$

When particle come to rest, $dx/dt = v = 0$

$\therefore\ 12 - 3t^2 = 0 \Rightarrow 3t^2 = 12 \Rightarrow t = 2$ sec.

Distance travelled by the particle before coming to rest

$$\int_0^s ds = \int_0^2 v dt \qquad s = \int_0^2 (12 - 3t^2)dt = 12t - \left.\frac{3t^3}{3}\right|_0^2$$

$s = 12 \times 2 - 8 = 24 - 8 = 16$ m.

22. (a) : $v^2 = u^2 - 2gh$

After reaching maximum height velocity becomes zero.

$$0 = (10)^2 - 2\times 10\times\frac{h}{2} \quad \therefore\ h = \frac{200}{20} = 10\text{ m}.$$

23. (d) : $x = ae^{-\alpha t} + be^{\beta t}$

$$\frac{dx}{dt} = -a\alpha e^{-\alpha t} + b\beta e^{\beta t}$$

$$v = -a\alpha e^{-\alpha t} + b\beta e^{\beta t}$$

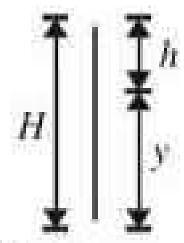
For certain value of t, velocity will increases.

24. (a) : Interval of ball thrown = 2 sec

If we want that minimum three (more than two) balls remain in air then time of flight of first ball must be greater than 4 sec.

$$T > 4\text{ sec or } \frac{2u}{g} > 4\text{ sec} \Rightarrow u > 19.6\text{ m/s}.$$

25. (b) : Let total height $= H$



Time of ascent $= T$

So, $H = uT - \frac{1}{2}gT^2$

Distance covered by ball in time $(T - t)$ sec.

$$y = u(T-t) - \frac{1}{2}g(T-t)^2$$

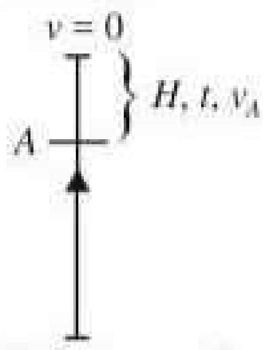
So distance covered by ball in last t sec.

$$h = H - y = \left[uT - \frac{1}{2}gT^2\right] - \left[u(T-t) - \frac{1}{2}g(T-t)^2\right]$$

By solving and putting $T = \frac{u}{g}$ we will get

$$h = \frac{1}{2}gt^2.$$

Aliter :



Time to reach the topmost position, $T = u/g$

Velocity at the top, $v = 0$

Let's consider a point A distance H below the highest point. Let it takes t seconds for the ball to reach the top from A. So we have to calculate H.

Let's find the velocity at point A. Now the time taken to reach A is $(T - t)$.

$\therefore\ v_A = u - g(T-t) = u - gT - gt = u - u - gt = -gt.$

Now consider its journey from A to the top.

Using $v^2 = u^2 - 2gh$

$$\Rightarrow\ 0 = v_A^2 - 2gH \Rightarrow H = \frac{(-gt)^2}{2g} = \frac{1}{2}gt^2.$$

26. (c) : For half height,

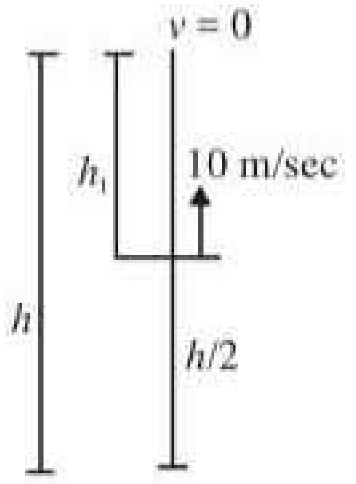
$10^2 = u^2 - 2g\frac{h}{2}$ ...(i)

For total height,

$0 = u^2 - 2gh$ ...(ii)

From (i) and (ii)

$\Rightarrow \quad 10^2 = \frac{2gh}{2} \Rightarrow h = 10$ m.



27. (b) : $\frac{ds}{dt} = 9t^2 + 14t + 14$

$\Rightarrow \quad \frac{d^2s}{dt^2} = 18t + 14 = a$

$a_{t=1} = 18 \times 1 + 14 = 32$ m/s^2

28. (c) : 1st case $v^2 - u^2 = 2as$

$0 - \left(\frac{100}{9}\right)^2 = 2 \times a \times 2$ [$\because$ 40 km/h = 100/9 m/s]

$a = -\frac{10^4}{81 \times 4}$ m/s

2nd case : $0 - \left(\frac{200}{9}\right)^2 = 2 \times \left(-\frac{10^4}{81 \times 4}\right) \times s$

[80 km/h = 200/9 m/s]

or $\quad s = 8$ m.

29. (a) : Initial energy equation

$mgh = \frac{1}{2}mv^2$ i.e. $10 \times 5 = \frac{1}{2}v_1^2 \Rightarrow v_1 = 10$ ...(i)

After one bounce, $10 \times 1.8 = \frac{1}{2}v_2^2 \Rightarrow v_2 = 6$ ...(ii)

Loss in velocity on bouncing $\frac{6}{10} = \frac{3}{5}$ a factor.

30. (a) : Distance $(x) = at^2 - bt^3$.

Therefore velocity $(v) = \frac{dx}{dt} = \frac{d}{dt}\left(at^2 - bt^3\right)$

$= 2at - 3bt^2$ and

acceleration $= \frac{dv}{dt} = \frac{d}{dt}\left(2at - 3bt^2\right) = 2a - 6bt = 0$

or $\quad t = \frac{2a}{6b} = \frac{a}{3b}$.

31. (d) : Initial velocity $u = 0$,

Final velocity = 144 km/h = 40 m/s and time = 20 sec.

Using $v = u + at \Rightarrow a = v/t = 2$ m/s^2

Again, $s = ut + \frac{1}{2}at^2 = \frac{1}{2} \times 2 \times (20)^2 = 400$ m.

32. (a) : Initial velocity of first body $(u_1) = 0$; Final velocity $(v_1) = 3$ m/s and initial velocity of second body $(u_2) = 4$ m/s.

height $(h) = \frac{v_1^2}{2g} = \frac{(3)^2}{2 \times 9.8} = 0.46$ m.

Therefore velocity of the second body,

$v_2 = \sqrt{u_2^2 + 2gh} = \sqrt{(4)^2 + 2 \times 9.8 \times 0.46} = 5$ m/s.

33. (c) : Acceleration μ bt. *i.e.*, $\frac{d^2x}{dt^2} = a \propto bt$

Integrating, $\frac{dx}{dt} = \frac{bt^2}{2} + C$

Initially, $t = 0$, $dx/dt = v_0$

Therefore, $\frac{dx}{dt} = \frac{bt^2}{2} + v_0$

Integrating again, $x = \frac{bt^3}{6} + v_0 t + C$

When $t = 0, x = 0 \Rightarrow C = 0$.

i.e., distance travelled by the particle in time t

$= v_0 t + \frac{bt^3}{6}$.

34. (a) : Height of tap = 5 m. For the first drop,

$5 = ut + \frac{1}{2}gt^2 = \frac{1}{2} \times 10t^2 = 5t^2$ or $t^2 = 1$

or $t = 1$ sec. It means that the third drop leaves after one second of the first drop, or each drop leaves after every 0.5 sec. Distance covered by the second drop in 0.5 sec

$= \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times (0.5)^2 = 1.25$ m.

Therefore distance of the second drop above the ground = 5 − 1.25 = 3.75 m.

35. (d) : Initial velocity $(u) = 0$; Acceleration in the first phase = α; Deceleration in the second phase = β and total time = t.

When car is accelerating then

final velocity $(v) = u + \alpha t = 0 + \alpha t_1$

or $t_1 = \frac{v}{\alpha}$ and when car is decelerating,

then final velocity $0 = v - \beta t$ or $t_2 = \frac{v}{\beta}$.

Therefore total time $(t) = t_1 + t_2 = \frac{v}{\alpha} + \frac{v}{\beta}$

$t = v\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) = v\left(\frac{\beta + \alpha}{\alpha\beta}\right)$ or $v = \frac{\alpha\beta t}{\alpha + \beta}$.

36. (c) : Displacement $(s) = t^3 - 6t^2 + 3t + 4$ metres.

velocity $(v) = \frac{ds}{dt} = 3t^2 - 12t + 3$

acceleration $(a) = \frac{dv}{dt} = 6t - 12$.

When $a = 0$, we get $t = 2$ seconds.

Therefore velocity when the acceleration is zero (v) $= 3 \times (2)^2 - (12 \times 2) + 3 = -9$ m/s.

37. (a) : Initial velocity (u) = 20 km/h; Final velocity (v) = 60 km/h and time (t) = 4 hours.

velocity (v) $= 60 = u + at = 20 + (a \times 4)$

or, $a = \dfrac{60-20}{4} = 10$ km/h^2.

Therefore distance travelled in 4 hours is s

$$s = ut + \frac{1}{2}at^2 = (20 \times 4) + \frac{1}{2} \times 10 \times (4)^2 = 160 \text{ km}.$$

38. (a) : The velocity $(v) = \dfrac{ds}{dt}$.

Therefore, instantaneous velocity at point E is negative.

39. (a) : Distance covered in n^{th} second is given by

$$s_n = u + \frac{a}{2}(2n-1)$$

Here, $u = 0$

$$\therefore \quad s_4 = 0 + \frac{a}{2}(2\times 4 - 1) = \frac{7a}{2}$$

$$s_3 = 0 + \frac{a}{2}(2\times 3 - 1) = \frac{5a}{2} \quad \therefore \quad \frac{s_4}{s_3} = \frac{7}{5}$$

40. (b) : In one dimensional motion, the body can have at a time one value of velocity but not two values of velocities.

41. (b) : Let h be height of the tower and t is the time taken by the body to reach the ground.

Here, $u = 0$, $a = g$

$$\therefore \quad h = ut + \frac{1}{2}gt^2 \text{ or } h = 0 \times t + \frac{1}{2}gt^2$$

or $h = \dfrac{1}{2}gt^2$ .....(i)

Distance covered in last two seconds is

$$40 = \frac{1}{2}gt^2 - \frac{1}{2}g(t-2)^2 \quad (\text{Here, } u = 0)$$

or $40 = \dfrac{1}{2}gt^2 - \dfrac{1}{2}g(t^2 + 4 - 4t)$

or $40 = (2t-2)g$ or $t = 3$ s

From eqn (i), we get $h = \dfrac{1}{2} \times 10 \times (3)^2$ or $h = 45$ m

42. (b) : Total distance travelled = 200 m

Total time taken $= \dfrac{100}{40} + \dfrac{100}{v}$

Average speed $= \dfrac{\text{total distance travelled}}{\text{total time taken}}$

$$48 = \frac{200}{\left(\frac{100}{40} + \frac{100}{v}\right)} \text{ or } 48 = \frac{2}{\left(\frac{1}{40} + \frac{1}{v}\right)}$$

or $\dfrac{1}{40} + \dfrac{1}{v} = \dfrac{1}{24}$

or $\dfrac{1}{v} = \dfrac{1}{24} - \dfrac{1}{40} = \dfrac{5-3}{120} = \dfrac{1}{60}$

or $v = 60$ km/hr

43. (c) : Total distance travelled = s

Total time taken $= \dfrac{s/3}{10} + \dfrac{s/3}{20} + \dfrac{s/3}{60}$

$$= \frac{s}{30} + \frac{s}{60} + \frac{s}{180} = \frac{10s}{180} = \frac{s}{18}$$

Average speed $= \dfrac{\text{total distance travelled}}{\text{total time taken}}$

$$= \frac{s}{s/18} = 18 \text{ km/hr.}$$

44. (b) : Total distance covered = s

Total time taken $= \dfrac{s/2}{40} + \dfrac{s/3}{60} = \dfrac{5s}{240} = \dfrac{s}{48}$

$\therefore$ Average speed $= \dfrac{\text{total distance covered}}{\text{total time taken}}$

$$= \frac{s}{\left(\frac{s}{48}\right)} = 48 \text{ km/hr}$$

45. (b) : Distance covered in n^{th} second is given by

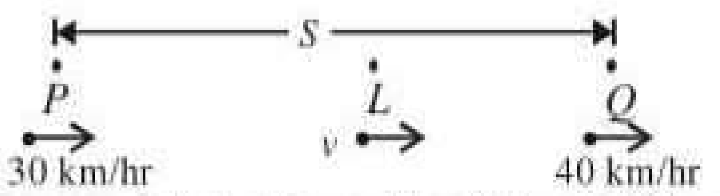
$$s_n = u + \frac{a}{2}(2n-1)$$

Given : $u = 0$, $a = g$

$$\therefore \quad s_4 = \frac{g}{2}(2\times 4 - 1) = \frac{7g}{2}$$

$$s_5 = \frac{g}{2}(2\times 5 - 1) = \frac{9g}{2} \quad \therefore \quad \frac{s_4}{s_5} = \frac{7}{9}$$

46. (c) :



Let $PQ = s$ and L is the midpoint of PQ and v be velocity f the car at point L.

Using third equation of motion, we get

$(40)^2 - (30)^2 = 2as$

or $a = \dfrac{(40)^2 - (30)^2}{2s} = \dfrac{350}{s}$ .....(i)

Also, $v^2 - (30)^2 = 2a\dfrac{s}{2}$

or $v^2 - (30)^2 = 2 \times \dfrac{350}{s} \times \dfrac{s}{2}$ [Using (i)]

or $v = 25\sqrt{2}$ km/hr